Node Voltage Method

The node voltage method provides a systematic means to specify the equations needed to **solve a circuit**. The term "solve a circuit" means to find all of the voltages and all of the currents for all of the components in the circuit.

The node voltage method uses the **essential nodes** in a circuit. Remember that the essential nodes are the points in the circuit where three or more circuit elements are connected.

The node voltage method uses **KCL** equations that are written at certain essential nodes. Recall that KCL states that the sum of all of the currents at a node is zero.

The node voltage method is comprised of the following steps:

1. Identify all of the essential nodes in the circuit. To do this we will place a large black dot at each essential node.

2. Choose one of the essential nodes as the reference node. We will use a special symbol to label the reference node.

3. Assign variable names to each of the non-reference essential nodes. Each variable name represents the voltage drop between its node and the reference node. We will use variable names like $v_1, v_\mathrm{a}, v_\Delta$ and so on.

4. Write a KCL equation at each of the non-reference essential nodes where the voltage with respect to the reference node is unknown. We will be methodical in writing these KCL equations, always summing the currents leaving the node.

5. Write any supplemental equations that are needed. These equations arise when there are dependent sources in the circuit, and when there are voltage sources in the circuit.

6. Express all of the equations in standard form. The standard form we use will allow our equations to be solved using a calculator, using a matrix method such as Cramer's rule, or using a computer tool such as MATLAB.

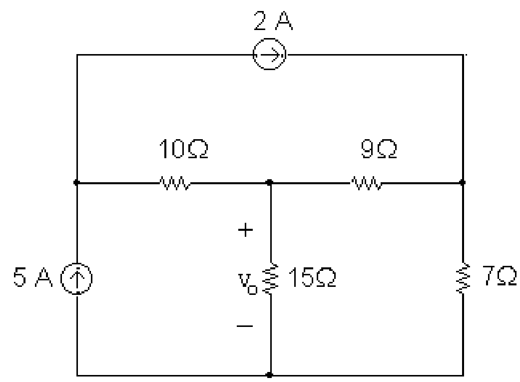

Figure 1: The circuit for Node Voltage Example 1

7. Solve the equations and check your solution using a power balance. If the power balances, use the solution to calculate the desired output value for the circuit.

We begin with an example that contains only resistors and independent current sources. Once you have mastered these types of circuits, we move on to example circuits containing dependent current sources, and then to circuits containing voltage sources.

Node Voltage Example 1

Using the node voltage method, find v_o for the circuit in Fig. 1.

Solution

1. Identify the essential nodes. There are four points at which three or more circuit elements connect, so there are four essential nodes. They have been labeled with large black dots in Fig. 2.

2. Chose a reference node. The choice of the reference node is entirely arbitrary; no matter which essential node is chosen, the voltages and currents that result from the analysis will have the same values. You should chose an essential node that makes the circuit analysis easier, if possible. In the circuit in Fig. 2 we have chosen the bottom node as the reference node, and labeled it with the symbol for circuit ground. We chose this node because it is one of the two nodes associated with v_o, the voltage of interest.

3. Assign variable names to the non-reference essential nodes. This is shown in Fig. 2. Note that we have labeled the center node v_o, because

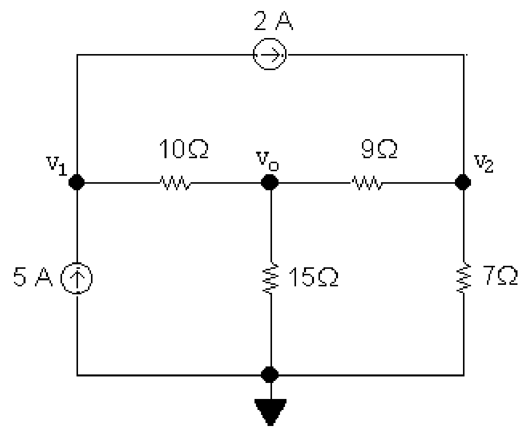

Figure 2: The circuit in Fig. 1 with the essential nodes marked, the reference node chosen, and the remaining essential nodes labeled.

it is the voltage we seek. Remember that these variable names represent the voltage difference between the node being labeled and the reference node.

4. Write a KCL equation at each non-reference essential node. We sum the currents leaving each node. The equations are given below. Note that since each node is a point at which three circuit components meet, each KCL equation has three terms.

$$\begin{aligned} \text{at } v_1: &\quad -5 + \frac{v_1 - v_o}{10} + 2 &= 0 \\ \text{at } v_o: &\quad \frac{v_o - v_1}{10} + \frac{v_o}{15} + \frac{v_o - v_2}{9} &= 0 \\ \text{at } v_2: &\quad \frac{v_2 - v_o}{9} + \frac{v_2}{7} - 2 &= 0 \end{aligned}$$

Note that we have three unknowns, the three node voltages v_o, v_1, and v_2, and three equations in terms of those unknowns.

5. Write any supplemental equations. In this example, there are no supplemental equations, since there are no dependent sources or voltage sources in the circuit. Also, we already have a sufficient number of equations to solve for all of the unknowns.

6. Express the equations in standard form. The form we use collects all of the terms involving each of the unknowns on the left-hand side of each equation, and collects the constants on the right-hand side of each

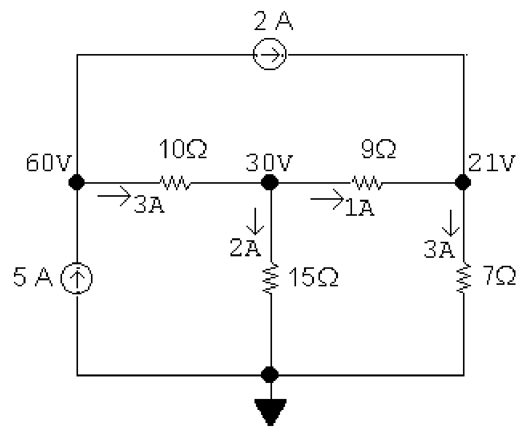

Figure 3: The circuit for Node Voltage Example 1, solved.

equation. This is shown below:

$$
\begin{array}{llllll}
\text{at } v_1: & v_1\left(\frac{1}{10}\right) & + \; v_o\left(-\frac{1}{10}\right) & + \; v_2\,(0) & = & 3 \\
\text{at } v_o: & v_1\left(-\frac{1}{10}\right) & + \; v_o\left(\frac{1}{10}+\frac{1}{15}+\frac{1}{9}\right) & + \; v_2\left(-\frac{1}{9}\right) & = & 0 \\
\text{at } v_2: & v_1\,(0) & + \; v_o\left(-\frac{1}{9}\right) & + \; v_2\left(\frac{1}{9}+\frac{1}{7}\right) & = & 2
\end{array}
$$

Note that there are three terms on the left-hand side of each equation, one for each of the three unknown variables. Be sure to check these equations against the KCL equations from the previous step to be certain you know how to use the standard form.

7. Solve the equations and check your solution. When these equations are input into a calculator, the solution is

$$v_1 = 60 \text{ V}; \qquad v_o = 30 \text{ V}; \qquad v_2 = 21 \text{ V}.$$

The circuit is repeated in Fig. 3 with the values of the node voltages labeled, and the currents through each of the branches labeled. Remember that we can calculate the current through each resistor using Ohm's law. Using the values in Fig. 3, we can calculate the power for each component:

$$
\begin{array}{lclclcl}
p_{5\text{A}} & = & -vi & = & -(60)(5) & = & -300 \text{ W;} \\
p_{2\text{A}} & = & vi & = & (60-21)(2) & = & 78 \text{ W;} \\
p_{10\Omega} & = & v^2/R & = & (60-30)^2/10 & = & 90 \text{ W;} \\
p_{15\Omega} & = & v^2/R & = & (30)^2/15 & = & 60 \text{ W;} \\
p_{9\Omega} & = & v^2/R & = & (30-21)^2/9 & = & 9 \text{ W;} \\
p_{7\Omega} & = & v^2/R & = & (21)^2/7 & = & 63 \text{ W;}
\end{array}
$$

Thus,

$$\sum p = -300 + 78 + 90 + 60 + 9 + 63 = 0 \text{ W} \qquad \text{checks}$$

The power balance verifies that we have the correct solution, so $v_o = 30$ V.

Now try using the node voltage method for each of the practice problems below.

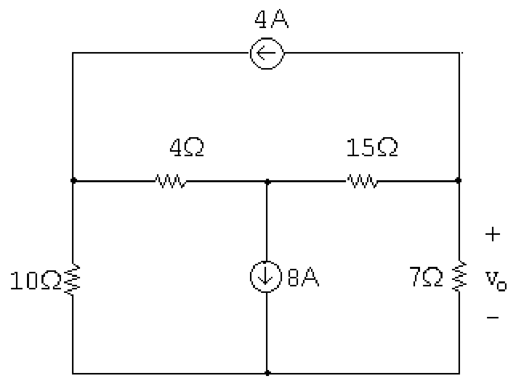

Figure 4: The circuit for Node Voltage Practice Problem 1.

Node Voltage Practice Problem 1

Find v_o for the circuit in Fig. 4.

1. Identify the essential nodes by adding black dots to Fig. 4.
2. Choose a reference node by adding the ground symbol to Fig. 4.
3. Assign variable names to the non-reference essential nodes in Fig. 4.
4. Write a KCL equation at each non-reference essential node.

5. Are any supplemental equations required? If not, why not? If so, write them in the space below.

6. Express all of the equations in standard form.

7. Solve the equations, using a calculator, a computer tool, or Cramer's method.

   Check your solution by calculating the power for each element and summing the power for all elements.

   Calculate v_o.

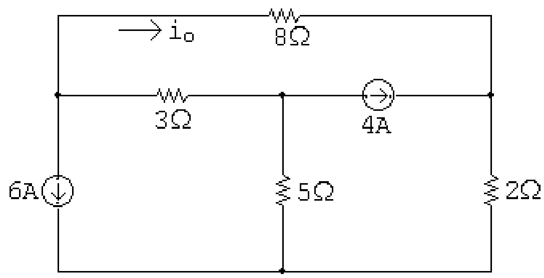

Figure 5: The circuit for Node Voltage Practice Problem 2.

Node Voltage Practice Problem 2

Find i_o for the circuit in Fig. 5.

1. Identify the essential nodes by adding black dots to Fig. 5.
2. Choose a reference node by adding the ground symbol to Fig. 5.
3. Assign variable names to the non-reference essential nodes in Fig. 5.
4. Write a KCL equation at each non-reference essential node.

5. Are any supplemental equations required? If not, why not? If so, write them in the space below.

6. Express all of the equations in standard form.

7. Solve the equations, using a calculator, a computer tool, or Cramer's method.

   Check your solution by calculating the power for each element and summing the power for all elements.

   Calculate i_o.

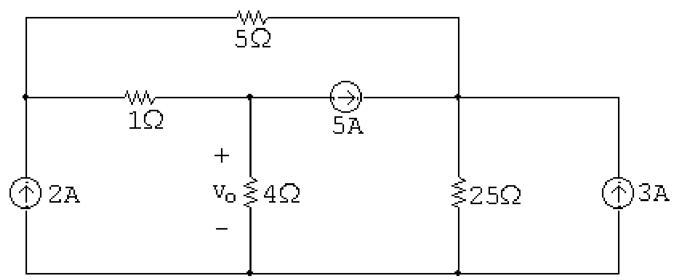

Figure 6: The circuit for Node Voltage Practice Problem 3.

Node Voltage Practice Problem 3

Find v_o for the circuit in Fig. 6.

1. Identify the essential nodes by adding black dots to Fig. 6.
2. Choose a reference node by adding the ground symbol to Fig. 6.
3. Assign variable names to the non-reference essential nodes in Fig. 6.
4. Write a KCL equation at each non-reference essential node.

5. Are any supplemental equations required? If not, why not? If so, write them in the space below.

6. Express all of the equations in standard form.

7. Solve the equations, using a calculator, a computer tool, or Cramer's method.

   Check your solution by calculating the power for each element and summing the power for all elements.

   Calculate v_o.

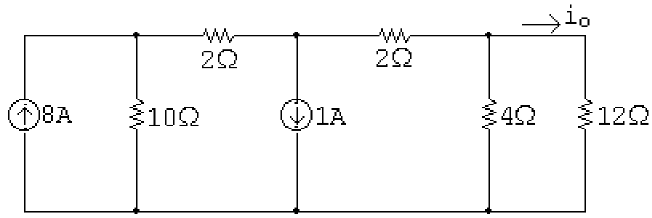

Figure 7: The circuit for Node Voltage Practice Problem 4.

Node Voltage Practice Problem 4

Find i_o for the circuit in Fig. 7.

1. Identify the essential nodes by adding black dots to Fig. 7.
2. Choose a reference node by adding the ground symbol to Fig. 7.
3. Assign variable names to the non-reference essential nodes in Fig. 7.
4. Write a KCL equation at each non-reference essential node.

5. Are any supplemental equations required? If not, why not? If so, write them in the space below.

6. Express all of the equations in standard form.

7. Solve the equations, using a calculator, a computer tool, or Cramer's method.

   Check your solution by calculating the power for each element and summing the power for all elements.

   Calculate i_o.

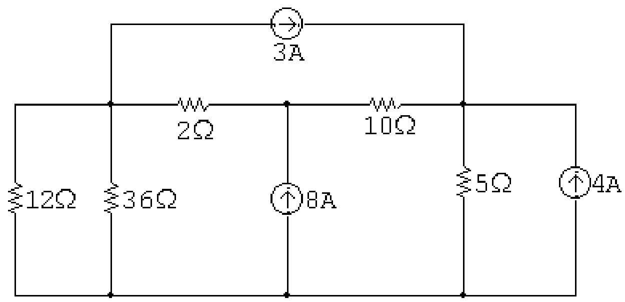

Figure 8: The circuit for Node Voltage Practice Problem 5.

Node Voltage Practice Problem 5

Find the power dissipated by the 10Ω resistor for the circuit in Fig. 8.

1. Identify the essential nodes by adding black dots to Fig. 8.

2. Choose a reference node by adding the ground symbol to Fig. 8.

3. Assign variable names to the non-reference essential nodes in Fig. 8.

4. Write a KCL equation at each non-reference essential node.

5. Are any supplemental equations required? If not, why not? If so, write them in the space below.

6. Express all of the equations in standard form.

7. Solve the equations, using a calculator, a computer tool, or Cramer's method.

   Check your solution by calculating the power for each element and summing the power for all elements.

   Calculate $p_{10\Omega}$.

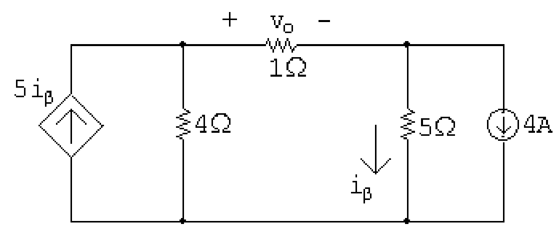

Figure 9: The circuit for Node Voltage Example 2

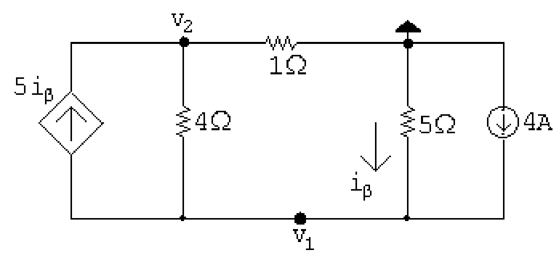

Figure 10: The circuit in Fig. 9 with the essential nodes marked, the reference node chosen, and the remaining essential nodes labeled.

Node Voltage Example 2

Using the node voltage method, find v_o for the circuit in Fig. 9

Solution

1. Identify the essential nodes. There are three points at which three or more circuit elements connect, so there are three essential nodes. They have been labeled with large black dots in Fig. 10.

2. Chose a reference node. In the circuit in Fig. 10 we have chosen the top right node as the reference node, and labeled it with the symbol for circuit ground. We chose this node because it is one of the two nodes associated with v_o, the voltage of interest.

3. Assign variable names to the non-reference essential nodes. This is shown in Fig. 10. Note that we have labeled the top left node v_o, because it is the voltage we seek. Remember that these variable names represent the voltage difference between the node being labeled and the reference node.

4. Write a KCL equation at each non-reference essential node. We sum

the currents leaving each node. The equations are given below.

$$\begin{aligned}
\text{at } v_o: \quad -5i_\beta + \frac{v_o - v_1}{4} + \frac{v_o}{1} &= 0 \\
\text{at } v_1: \quad 5i_\beta + \frac{v_1 - v_o}{4} + \frac{v_1}{5} - 4 &= 0
\end{aligned}$$

Note that we have three unknowns, the two node voltages v_o and v_1, and the current i_β that controls the dependent source. Yet we only have two KCL equations. This means we have to specify a third equation.

5. Write any supplemental equations. This is where the third equation will be developed. Whenever there are dependent sources in our circuit, we will need to write a supplemental equation that defines the voltage or current used to control the dependent source in terms of the node voltages in our circuit. This supplemental equation is also called a **constraint equation**, because it constrains the relationship between two or more unknowns in our circuit, so that one of the unknowns is no longer an independent variable but rather is dependent on the other independent variables in our circuit.

   Notice that the controlling current i_β is the current through the $5\,\Omega$ resistor, so we use Ohm's law to define this current in terms of the voltage difference across the resistor and the resistance. The constraint equation is thus

$$i_\beta = \frac{0 - v_1}{5}$$

   The two KCL equations and this constraint equation now provide the three equations needed to solve for the three unknowns in the circuit.

6. Express the equations in standard form. This is shown below:

$$\begin{aligned}
\text{at } v_o: &\quad i_\beta\,(-5) + v_1\left(-\frac{1}{4}\right) + v_o\left(\frac{1}{4}+1\right) = 0 \\
\text{at } v_1: &\quad i_\beta\,(5) + v_1\left(\frac{1}{4}+\frac{1}{5}\right) + v_o\left(-\frac{1}{4}\right) = 4 \\
\text{constraint: } &\quad i_\beta\,(1) + v_1\left(\frac{1}{5}\right) + v_o\,(0) = 0
\end{aligned}$$

7. Solve the equations and check your solution. When these equations are input into a calculator, the solution is

$$i_\beta = 2\text{ A}; \qquad v_1 = -10\text{ V}; \qquad v_o = 6\text{ V}.$$

   The circuit is repeated in Fig. 11 with the values of the node voltages labeled, and the currents through each of the branches labeled. Remember that we can calculate the current through each resistor using

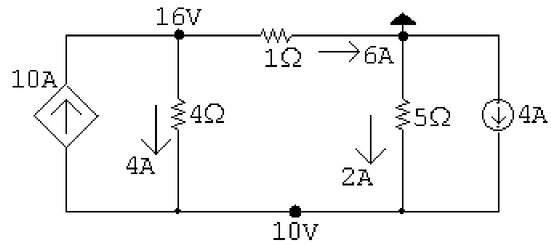

Figure 11: The circuit for Node Voltage Example 2, solved.

Ohm's law. Using the values in Fig. 11, we can calculate the power for each component:

$$
\begin{aligned}
p_{5i_\beta} &= vi &&= (-10-6)[5(2)] &&= -160 \text{ W};\\
p_{4\text{A}} &= vi &&= [0-(-10)](4) &&= 40 \text{ W};\\
p_{4\Omega} &= v^2/R &&= [6-(-10)]^2/4 &&= 64 \text{ W};\\
p_{1\Omega} &= v^2/R &&= (6-0)^2/1 &&= 36 \text{ W};\\
p_{5\Omega} &= v^2/R &&= (-10-0)^2/5 &&= 20 \text{ W};
\end{aligned}
$$

Thus,

$$\sum p = -160 + 40 + 64 + 36 + 20 = 0 \text{ W} \qquad \text{checks}$$

The power balance verifies that we have the correct solution, so $v_o = 6$ V.

Now try using the node voltage method as it applies to circuits with dependent sources for each of the practice problems below.

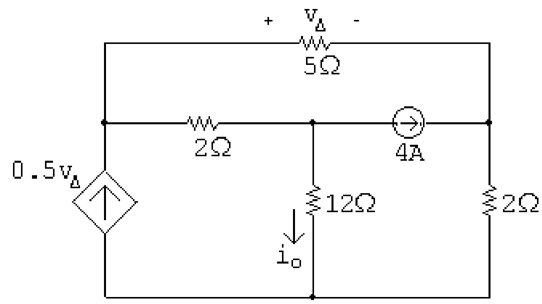

Figure 12: The circuit for Node Voltage Practice Problem 6.

Node Voltage Practice Problem 6

Find i_o for the circuit in Fig. 12.

1. Identify the essential nodes by adding black dots to Fig. 12.

2. Choose a reference node by adding the ground symbol to Fig. 12.

3. Assign variable names to the non-reference essential nodes in Fig. 12.

4. Write a KCL equation at each non-reference essential node.

5. Are any supplemental equations required? If not, why not? If so, write them in the space below.

6. Express all of the equations in standard form.

7. Solve the equations, using a calculator, a computer tool, or Cramer's method.

   Check your solution by calculating the power for each element and summing the power for all elements.

   Calculate i_o.

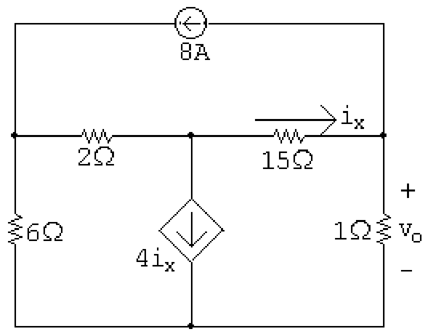

Figure 13: The circuit for Node Voltage Practice Problem 7.

Node Voltage Practice Problem 7

Find v_o for the circuit in Fig. 13.

1. Identify the essential nodes by adding black dots to Fig. 13.

2. Choose a reference node by adding the ground symbol to Fig. 13.

3. Assign variable names to the non-reference essential nodes in Fig. 13.

4. Write a KCL equation at each non-reference essential node.

5. Are any supplemental equations required? If not, why not? If so, write them in the space below.

6. Express all of the equations in standard form.

7. Solve the equations, using a calculator, a computer tool, or Cramer's method.

   Check your solution by calculating the power for each element and summing the power for all elements.

   Calculate v_o.

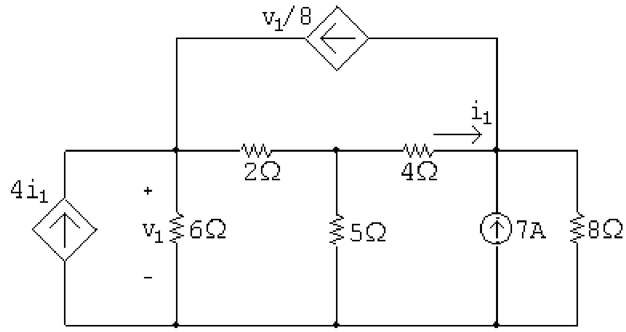

Figure 14: The circuit for Node Voltage Practice Problem 8.

Node Voltage Practice Problem 8

Find the power delivered to the circuit in Fig. 14.

1. Identify the essential nodes by adding black dots to Fig. 14.

2. Choose a reference node by adding the ground symbol to Fig. 14.

3. Assign variable names to the non-reference essential nodes in Fig. 14.

4. Write a KCL equation at each non-reference essential node.

5. Are any supplemental equations required? If not, why not? If so, write them in the space below.

6. Express all of the equations in standard form.

7. Solve the equations, using a calculator, a computer tool, or Cramer's method.

   Check your solution by calculating the power for each element and summing the power for all elements.

   Calculate the power delivered to the circuit.

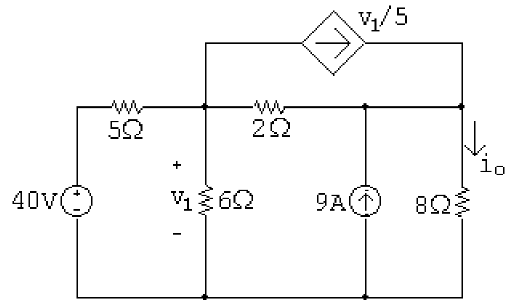

Figure 15: The circuit for Node Voltage Practice Problem 9.

Node Voltage Practice Problem 9

Find i_o for the circuit in Fig. 15.

1. Identify the essential nodes by adding black dots to Fig. 15.

2. Choose a reference node by adding the ground symbol to Fig. 15.

3. Assign variable names to the non-reference essential nodes in Fig. 15.

4. Write a KCL equation at each non-reference essential node.

5. Are any supplemental equations required? If not, why not? If so, write them in the space below.

6. Express all of the equations in standard form.

7. Solve the equations, using a calculator, a computer tool, or Cramer's method.

   Check your solution by calculating the power for each element and summing the power for all elements.

   Calculate i_o.

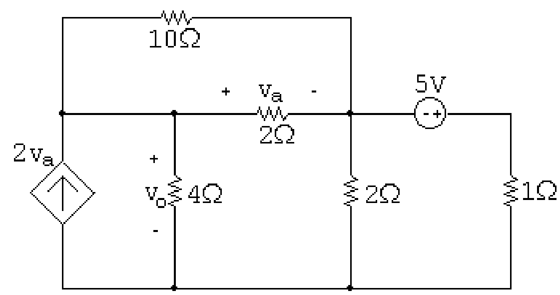

Figure 16: The circuit for Node Voltage Practice Problem 10.

Node Voltage Practice Problem 10

Find v_o for the circuit in Fig. 16.

1. Identify the essential nodes by adding black dots to Fig. 16.

2. Choose a reference node by adding the ground symbol to Fig. 16.

3. Assign variable names to the non-reference essential nodes in Fig. 16.

4. Write a KCL equation at each non-reference essential node. (*Hint* — interchange the positions of the 5 V source and the 2 Ω resistor.)

5. Are any supplemental equations required? If not, why not? If so, write them in the space below.

6. Express all of the equations in standard form.

7. Solve the equations, using a calculator, a computer tool, or Cramer's method.

   Check your solution by calculating the power for each element and summing the power for all elements.

   Calculate v_o.

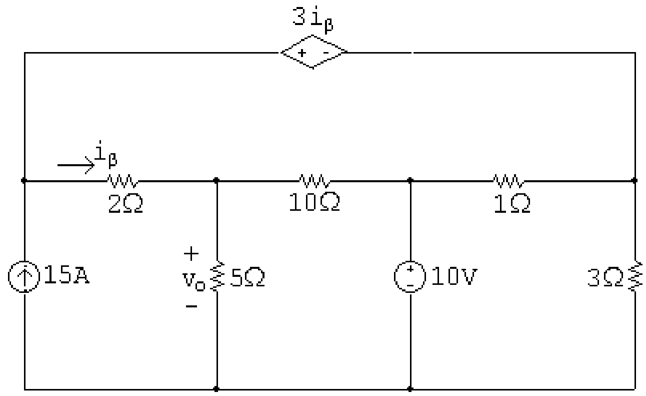

Figure 17: The circuit for Node Voltage Example 3

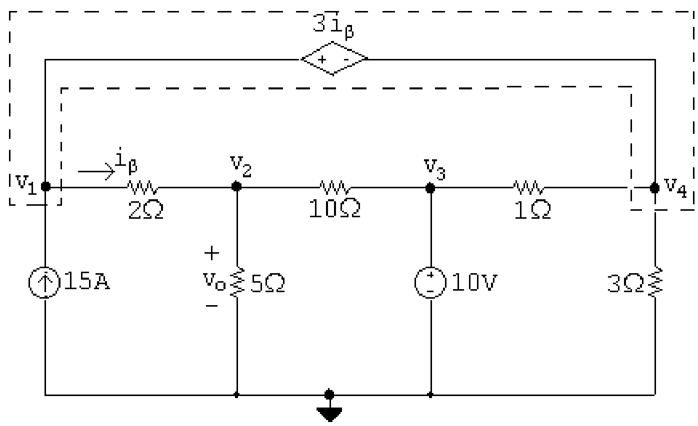

Figure 18: The circuit in Fig. 17 with the essential nodes marked, the reference node chosen, the remaining essential nodes labeled, and the supernode identified.

Node Voltage Example 3

Using the node voltage method, find v_o for the circuit in Fig. 17

Solution

1. Identify the essential nodes. There are five points at which three or more circuit elements connect, so there are five essential nodes. They have been labeled with large black dots in Fig. 18.

2. Chose a reference node. In the circuit in Fig. 18 we have chosen the top right node as the reference node, and labeled it with the symbol for circuit ground. We chose this node because it is one of the two nodes associated with v_o, the voltage of interest.

3. Assign variable names to the non-reference essential nodes. This is shown in Fig. 18. Note that the node labeled v_2 could also have been

labeled v_o, as this is the node that defines the desired output with respect to the reference node.

4. Write a KCL equation at each non-reference essential node. We modify this step whenever the circuit has a voltage source between two essential nodes. This circuit has two such voltage sources. Consider first the 10 V source. Since this voltage source is between a non-reference essential node (the node labeled v_3) and the reference node, it establishes a voltage of 10 V at the non-reference essential node. Thus, $v_3 = 10$ V, so there is no need to write a KCL equation at the node labeled v_3.

   Now consider the dependent voltage source. It, too, is between two essential nodes, but now neither node is the reference node. Any time a voltage source is between two non-reference essential nodes, it constrains the difference between the two voltages and forms a **supernode**. To deal with the supernode, we write one KCL equation for the supernode, and one constraint equation defining the relationship between the two node voltages that comprise the supernode. The supernode is identified by the dashed area in Fig. 18.

   Thus, in this step we write a KCL equation at each non-reference essential node whose voltage is not known, and at each supernode. For the circuit in Fig. 18, there is one known node voltage, two node voltages that comprise the supernode, and one remaining unknown node voltage. Thus we write two KCL equations, given below.

$$\begin{aligned} \text{at } v_2: \quad & \frac{v_2 - v_1}{2} + \frac{v_2}{5} + \frac{v_2 - 10}{10} &= 0 \\ \text{at supernode:} \quad & -15 + \frac{v_1 - v_2}{2} + \frac{v_4 - 10}{1} + \frac{v_4}{3} &= 0 \end{aligned}$$

5. Write any supplemental equations. Since there is a dependent source in the circuit, we know we will need at least one supplemental equation. This equation defines the quantity used to control the dependent source, i_β in terms of the labeled node voltages. Thus, the equation is

$$i_\beta = \frac{v_1 - v_2}{2}$$

   But we are not finished yet! Remember that the existence of a supernode means that two of the essential nodes are constrained by the voltage source in between these nodes. Therefore, every time we define a supernode in a circuit, we should expect to write a supplemental, or constraint, equation that relates the two essential nodes contained by the supernode. In this circuit, the supernode contains the essential

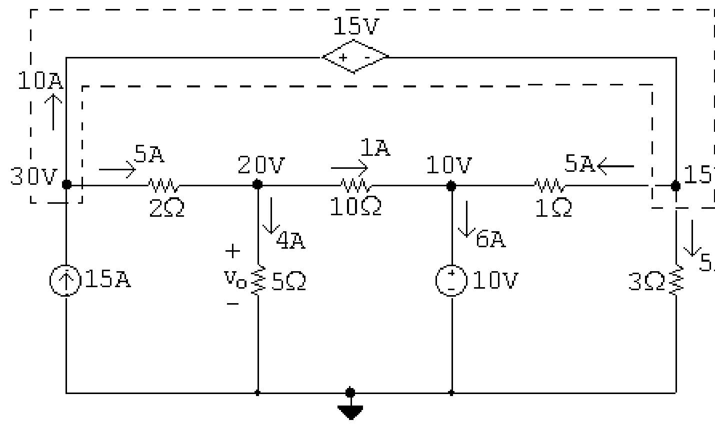

Figure 19: The circuit for Node Voltage Example 3, solved.

nodes labeled v_1 and v_4, and the constraint equation defines the limitation on the voltage difference between these two nodes. The constraint equation is

$$3i_\beta = v_1 - v_4$$

The two KCL equations and the two supplemental equations provide the four independant equations needed to solve for our three unknown essential node voltages (v_1, v_2, and v_4 — remember that $v_3 = 10$ V because of the independent voltage source between v_3 and the reference node) and our unknown controlling current (i_β).

6. Express the equations in standard form. This is shown below:

$$\begin{array}{llllllllll}
\text{at } v_2: & v_1\left(-\frac{1}{2}\right) & + & v_2\left(\frac{1}{2}+\frac{1}{5}+\frac{1}{10}\right) & + & v_4(0) & + & i_\beta(0) & = & 1 \\
\text{at supernode:} & v_1\left(\frac{1}{2}\right) & + & v_2\left(-\frac{1}{2}\right) & + & v_4\left(1+\frac{1}{3}\right) & + & i_\beta(0) & = & 25 \\
\text{dep. source:} & v_1\left(-\frac{1}{2}\right) & + & v_2\left(\frac{1}{2}\right) & + & v_4(0) & + & i_\beta(1) & = & 0 \\
\text{supernode:} & v_1(-1) & + & v_2(0) & + & v_4(1) & + & i_\beta(3) & = & 0
\end{array}$$

7. Solve the equations and check your solution. When these equations are input into a calculator, the solution is

$$v_1 = 30 \text{ V}; \qquad v_2 = 20 \text{ V}; \qquad v_4 = 15 \text{ V}; \qquad i_\beta = 5\text{A}.$$

The circuit is repeated in Fig. 19 with the values of the node voltages labeled, and the currents through each of the branches labeled. Remember that we can calculate the current through each resistor using Ohm's law. Using the values in Fig. 19, we can calculate the power for each component:

$$
\begin{array}{lllllll}
p_{15\text{A}} & = & -vi & = & -(15)(30) & = & -450 \text{ W}; \\
p_{\text{d.s.}} & = & vi & = & [3(5)](10) & = & 150 \text{ W}; \\
p_{10\text{V}} & = & vi & = & (6)(10) & = & 60 \text{ W}; \\
p_{2\Omega} & = & i^2R & = & (5)^2(2) & = & 50 \text{ W}; \\
p_{5\Omega} & = & i^2R & = & (4)^2(5) & = & 80 \text{ W}; \\
p_{10\Omega} & = & i^2R & = & (1)^2(10) & = & 10 \text{ W}; \\
p_{1\Omega} & = & i^2R & = & (5)^2(1) & = & 25 \text{ W}; \\
p_{3\Omega} & = & i^2R & = & (5)^2(3) & = & 75 \text{ W};
\end{array}
$$

Thus,

$$\sum p = -450 + 150 + 60 + 50 + 80 + 10 + 25 + 75 = 0 \text{ W} \qquad \text{checks}$$

The power balance verifies that we have the correct solution, so $v_o = 20$ V.

Now try using the node voltage method as it applies to circuits with voltage sources between essential nodes for each of the practice problems below.

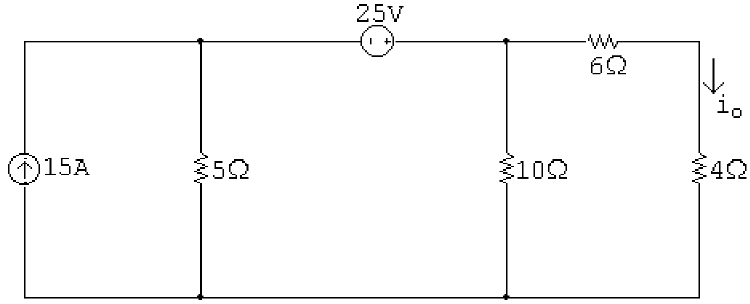

Figure 20: The circuit for Node Voltage Practice Problem 11.

Node Voltage Practice Problem 11

Find i_o for the circuit in Fig. 20.

1. Identify the essential nodes by adding black dots to Fig. 20.

2. Choose a reference node by adding the ground symbol to Fig. 20.

3. Assign variable names to the non-reference essential nodes in Fig. 20.

4. Write a KCL equation at each non-reference essential node for which the voltage is not already known, and at each supernode.

5. Are any supplemental equations required? If not, why not? If so, write them in the space below.

6. Express all of the equations in standard form.

7. Solve the equations, using a calculator, a computer tool, or Cramer's method.

   Check your solution by calculating the power for each element and summing the power for all elements.

   Calculate i_o.

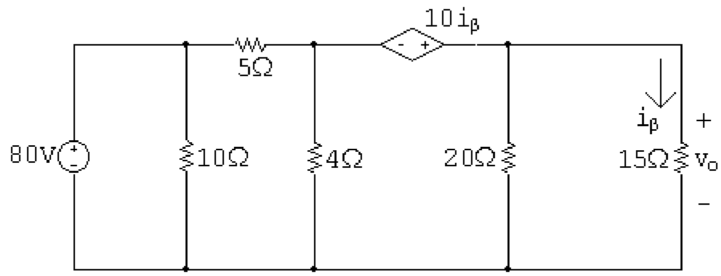

Figure 21: The circuit for Node Voltage Practice Problem 12.

Node Voltage Practice Problem 12

Find v_o for the circuit in Fig. 21.

1. Identify the essential nodes by adding black dots to Fig. 21.
2. Choose a reference node by adding the ground symbol to Fig. 21.
3. Assign variable names to the non-reference essential nodes in Fig. 21.
4. Write a KCL equation at each non-reference essential node for which the voltage is not already known, and at each supernode.

5. Are any supplemental equations required? If not, why not? If so, write them in the space below.

6. Express all of the equations in standard form.

7. Solve the equations, using a calculator, a computer tool, or Cramer's method.

   Check your solution by calculating the power for each element and summing the power for all elements.

   Calculate v_o.

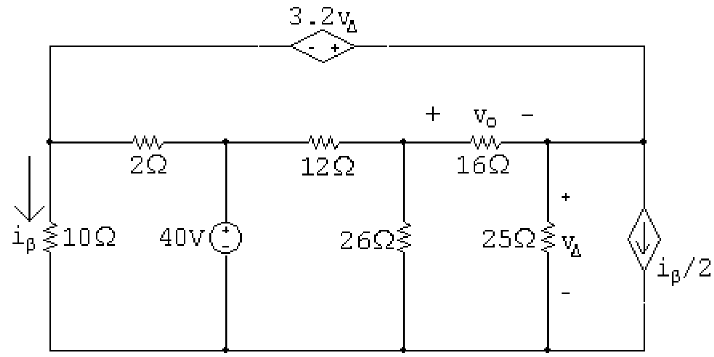

Figure 22: The circuit for Node Voltage Practice Problem 13.

Node Voltage Practice Problem 13

Find v_o for the circuit in Fig. 22.

1. Identify the essential nodes by adding black dots to Fig. 22.

2. Choose a reference node by adding the ground symbol to Fig. 22.

3. Assign variable names to the non-reference essential nodes in Fig. 22.

4. Write a KCL equation at each non-reference essential node for which the voltage is not already known, and at each supernode.

5. Are any supplemental equations required? If not, why not? If so, write them in the space below.

6. Express all of the equations in standard form.

7. Solve the equations, using a calculator, a computer tool, or Cramer's method.

   Check your solution by calculating the power for each element and summing the power for all elements.

   Calculate v_o.

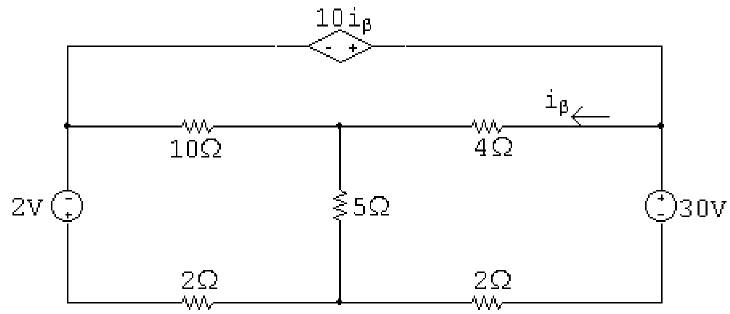

Figure 23: The circuit for Node Voltage Practice Problem 14.

Node Voltage Practice Problem 14

Find the power delivered to the circuit in Fig. 23.

1. Identify the essential nodes by adding black dots to Fig. 23.

2. Choose a reference node by adding the ground symbol to Fig. 23.

3. Assign variable names to the non-reference essential nodes in Fig. 23.

4. Write a KCL equation at each non-reference essential node for which the voltage is not already known, and at each supernode.

5. Are any supplemental equations required? If not, why not? If so, write them in the space below.

6. Express all of the equations in standard form.

7. Solve the equations, using a calculator, a computer tool, or Cramer's method.

   Check your solution by calculating the power for each element and summing the power for all elements.

   Calculate the power delivered to the circuit.

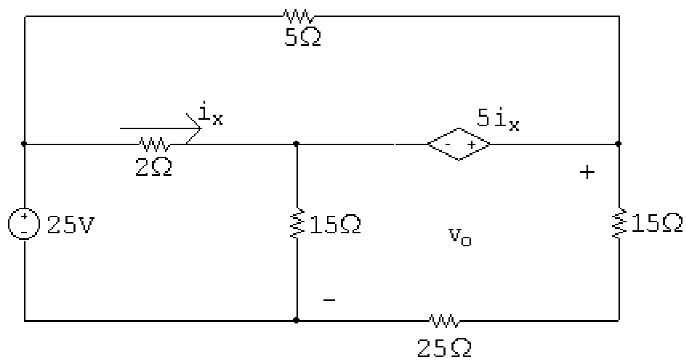

Figure 24: The circuit for Node Voltage Practice Problem 15.

Node Voltage Practice Problem 15

Find v_o for the circuit in Fig. 24.

1. Identify the essential nodes by adding black dots to Fig. 24.
2. Choose a reference node by adding the ground symbol to Fig. 24.
3. Assign variable names to the non-reference essential nodes in Fig. 24.
4. Write a KCL equation at each non-reference essential node for which the voltage is not already known, and at each supernode.

5. Are any supplemental equations required? If not, why not? If so, write them in the space below.

6. Express all of the equations in standard form.

7. Solve the equations, using a calculator, a computer tool, or Cramer's method.

   Check your solution by calculating the power for each element and summing the power for all elements.

   Calculate v_o.

Reading

- in *Introductory Circuits for Electrical and Computer Engineering*:
  - Section 3.1 — terminology and definitions
  - Section 3.2 — introduction to node voltage method
  - Section 3.3 — node voltage method with circuits containing dependent sources
  - Section 3.4 — supernodes
- in *Electric Circuits*, sixth edition:
  - Section 4.1 — terminology and definitions
  - Section 4.2 — introduction to node voltage method
  - Section 4.3 — node voltage method with circuits containing dependent sources
  - Section 4.4 — supernodes
- Workbook section — Power Balancing in DC Circuits

Additional Problems

- in *Introductory Circuits for Electrical and Computer Engineering*:
  - 3.2 — 3.13
  - 3.16 — 3.23
  - 3.25
- in *Electric Circuits*, sixth edition:
  - 4.2 — 4.13
  - 4.16 — 4.18
  - 4.20 — 4.26

Solutions

- Node Voltage Practice Problem 1 — with the lower node chosen as the reference node, the node voltages are 70V, 82V, and 7V and $v_o = 7$V.
- Node Voltage Practice Problem 2 — with the lower node chosen as the reference node, the node voltages are 38V, 65V, and 14V and $i_o = 3$A.
- Node Voltage Practice Problem 3 — with the lower node chosen as the reference node, the node voltages are 20V, 12V, and 50V and $v_o = 12$V.

- Node Voltage Practice Problem 4 — with the lower node chosen as the reference node, the node voltages are 30V, 20V, and 12V and $i_o = 1$A.
- Node Voltage Practice Problem 5 — with the lower node chosen as the reference node, the node voltages are 36V, 50V, and 40V and $p_1 0\Omega =$ 10W.
- Node Voltage Practice Problem 6 — with the lower node chosen as the reference node, the node voltages are 36V, 24V, and 16V and $i_o = 2$A.
- Node Voltage Practice Problem 7 — with the lower node chosen as the reference node, the node voltages are 18V, 8V, and −7V and $v_o = -7$V.
- Node Voltage Practice Problem 8 — with the lower node chosen as the reference node, the node voltages are 64V, 40V, and 24V and $p_{\text{delivered}} =$ 1000W.
- Node Voltage Practice Problem 9 — with the lower node chosen as the reference node, the node voltages are 30V and 48V and $i_o = 6$A.
- Node Voltage Practice Problem 10 — with the lower node chosen as the reference node, the node voltages are 20V and 10V and $v_o = 20$V.
- Node Voltage Practice Problem 11 — with the lower node chosen as the reference node, the node voltages are 25V and 50V and $i_o = 5$A.
- Node Voltage Practice Problem 12 — with the lower node chosen as the reference node, the node voltages are 80V, 20V, and 60V and $v_o = 60$V.
- Node Voltage Practice Problem 13 — with the lower node chosen as the reference node, the node voltages are 20V, 40V, 52V, and 100V and $v_o = -48$V.
- Node Voltage Practice Problem 14 — with the lower node chosen as the reference node, the node voltages are −40V, 20V, and 60V and $p_{\text{delivered}} = 2500$V.
- Node Voltage Practice Problem 15 — with the lower node chosen as the reference node, the node voltages are 25V, 15V, and 40V and $v_o = 40$V.